Measures of Center and Spread

Lesson 36-1 · adapted

Mean, Median, Mode, and MAD

The Scout's Contract · a statistics lesson · Designed for a 75-minute class · Grade 9 Algebra 1

Noah Libby-Haines

WHAT WE'LL LEARN

By the end of this lesson

1

Center

How the center of a dataset can be defined mathematically, and how to calculate it three ways: mean, median, and mode.

2

Spread

Why the center alone can hide the story, how to calculate range and MAD, and what a MAD means in context.

3

Decide

How to compare data sets using center and spread together to justify a decision, and to question what a statistic really measures.

STAGE 1 · THE QUESTION

You are running a football club.

Your club can sign only one striker. **Hùng** and **Đức** are both available, and they cost the same. Here are their goals in the last five months:

Goals	Aug	Sep	Oct	Nov	Dec
Hùng	6	8	2	0	4
Đức	4	4	4	4	4

Who do you sign? HÙNG or ĐỨC?

On your own, write: Hùng or Đức, and one reason. Use only the goals in the table. No calculations yet.

Then compare with your partner. Keep your answer: we come back to it later.

Assume: every striker plays every match, full minutes, in the same league.

Football words

Striker: the player whose main job is to score goals.

Sign: give a player a contract with your club.

STAGE 1 · THE QUESTION

Same total. Same player?

Hands up: who chose **Hùng**? Who chose **Đức**?
Then, with your partner, add up each striker's goals.

Goals	Aug	Sep	Oct	Nov	Dec	Total
Hùng	6	8	2	0	4	20
Đức	4	4	4	4	4	20

Same total goals. But the class did not all choose the same striker. What is different about their goals?

Discuss

Keep that question. First, we need one number that describes a striker's goals: the average.

STAGE 2 · MEASURING CENTER

Finding the mean

Goals	Aug	Sep	Oct	Nov	Dec
Hùng	6	8	2	0	4
Đức	4	4	4	4	4

Hùng: $(6 + 8 + 2 + 0 + 4) \div 5 = 20 \div 5 = 4$

Đức: $(4 \times 5) \div 5 = 20 \div 5 = 4$

Mean = add every value, then divide by how many there are.

Both strikers average 4 goals a month.

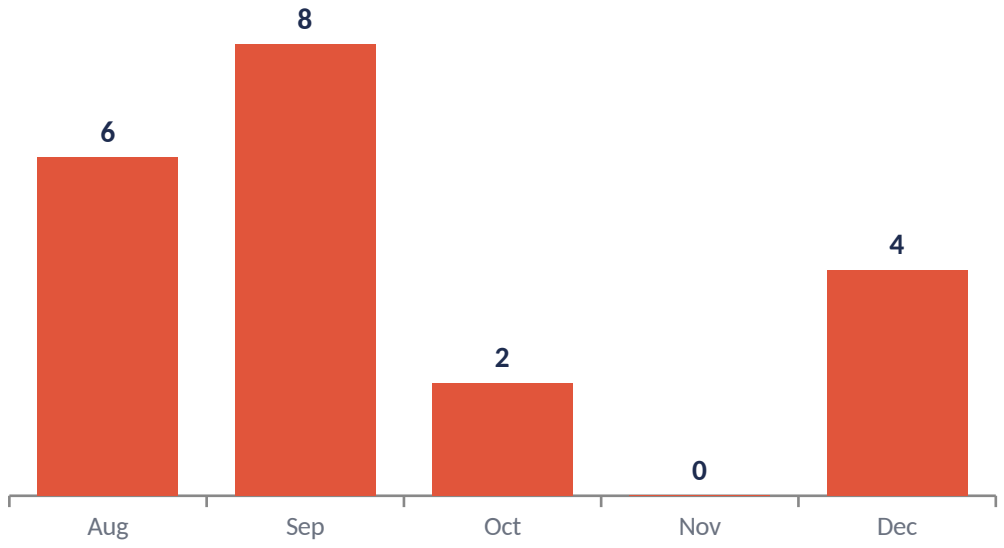
STAGE 2 · MEASURING CENTER

Same mean. So what does it tell us?

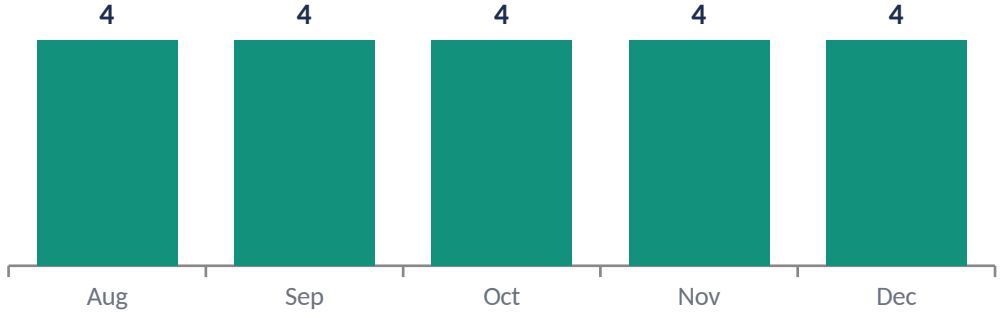
Both average 4 goals a month. Do they score the same way? How are they alike? How are they different?

Discuss · 60 seconds

Hùng



Đức



STAGE 2 · MEASURING CENTER

Median: the middle value

Hùng's goals, sorted: 0, 2, **4**, 6, 8 — Median = 4

Đức's goals, sorted: 4, 4, **4**, 4, 4 — Median = 4

Sort

Put the values in order, smallest to largest.

Odd count

The median is the middle value.

Even count

Average the two middle values.

STAGE 2 · MEASURING CENTER

Mode: the most repeated value

Hùng: 0, 2, 4, 6, 8 (every value appears once) → **Mode: not defined**

Đức: 4, 4, 4, 4, 4 (4 appears every time) → **Mode = 4**

Count repeats

Tally how many times each value appears.

One winner

If one value repeats most, that is the mode.

Ties / not defined

Several can tie; if nothing repeats, the mode is not defined.

STAGE 2 · SPEED SCOUTING

Whiteboard Race

Rules

Solve the problems:

- Work with your partner.
- Write your answer.
- Be ready to show your work.
- When you both understand, show your answer.

Be ready to explain your work.

Points for teams that get the correct answer and can explain how.

Problem 1

Football · Median

A team's goals in its last 6 matches:

0, 1, 1, 3, 4, 6

Find the median.

Median = 2 goals

Two middle values: (1 + 3) ÷ 2 = 2

Problem 2

Football · Mode

A striker's shots in his last 7 matches:

3, 5, 2, 5, 8, 5, 1

Find the mode.

Mode = 5 shots

5 appears 3 times

Problem 3

Free Fire · Median

A player's kills in her last 7 matches:

6, 2, 9, 4, 0, 7, 4

Find the median.

In order: 0, 2, 4, 4, 6, 7, 9

Median = 4 kills

The middle (4th) of 7 values

Problem 4

Free Fire · Median and mode

A player's kills in her last 8 matches:

7, 2, 5, 9, 7, 3, 1, 7

Find the median AND the mode.

In order: 1, 2, 3, 5, 7, 7, 7, 9

Median = 6 kills · Mode = 7 kills

(5 + 7) ÷ 2 = 6 · 7 appears 3 times

STAGE 2 · MEASURING CENTER

What do the measures of center say?

Goals	Aug	Sep	Oct	Nov	Dec
Hùng	6	8	2	0	4
Đức	4	4	4	4	4

On your whiteboard, find the mean, median and mode for both strikers.

	Hùng	Đức
Mean	4	4
Median	4	4
Mode	Mode: not defined	4

Look at the goals and the table.

1. What is the same about Hùng and Đức?
2. What is different about their goals each month?
3. Does the table show that difference?

Discuss

The mean and the median are the same for both strikers; the mode is different. **But none of these measures shows how spread out the months are:** Hùng's months are spread out, and Đức's are all the same.

STAGE 3 · MEASURING SPREAD

Free Fire: Nam and Thảo

Kills	Match 1	Match 2	Match 3	Match 4	Match 5	Match 6
Nam	5	6	1	7	11	6
Thảo	1	11	3	9	1	11

How are Nam and Thảo different?

With your partner **Calculate the mean and the range for both players. What do you notice?**

Mean = sum ÷ n

Nam: 36 ÷ 6 = **6** Thảo: 36 ÷ 6 = **6**

Range = max − min

Nam: 11 − 1 = **10** Thảo: 11 − 1 = **10**

Same mean. Same range. Are their matches similar? What does the range tell us? What does it not tell us?

Discuss

STAGE 3 · MEASURING SPREAD

The problem with range

Nam:	1, 5, 6, 6, 7, 11	*most matches close to 6*	→ mean 6, range 10
Thảo:	1, 1, 3, 9, 11, 11	*no match close to 6*	→ mean 6, range 10

Same range, but Thảo's matches are all far from 6, while most of Nam's are close to 6. Range only looks at the two ends and ignores everything in between.

How could we measure spread using **every** value, not only the highest and the lowest? What could we measure for each match?

Discuss

STAGE 4 · MEAN ABSOLUTE DEVIATION

A measure that uses every value: MAD

Core idea: how far is each value from the center? MAD is the average of those distances.

1

Find the mean

Calculate the average. This is your center.

2

Find each deviation

x − mean is the deviation from the mean. It can be negative.

3

Average the distances

Use |x − mean| so negatives don't cancel, then take the mean.

MAD = the average distance of every value from the mean.

STAGE 4 · MEAN ABSOLUTE DEVIATION

Step by step: Hùng's MAD

With your partner: complete the table on your whiteboard

Goals (x)	Deviation x − mean	Distance |x − mean|
6		
8		
2		
0		
4		
Sum = ?	**Sum = ?**	**Sum = ?**

Work out:

1. Find the total of the goals.
2. Find the mean.
3. Fill in the table.
4. MAD = (sum of distances) ÷ 5

Watch the middle column. What do the deviations add up to?

STAGE 4 · MEAN ABSOLUTE DEVIATION

Hùng's MAD: the answers

Total = 6 + 8 + 2 + 0 + 4 = 20 **Mean = 20 ÷ 5 = 4**

Goals (x)	Deviation x − mean	Distance |x − mean|
6	6 − 4 = +2	2
8	8 − 4 = +4	4
2	2 − 4 = −2	2
0	0 − 4 = −4	4
4	4 − 4 = 0	0
Sum = 20	**Sum = 0**	**Sum = 12**

Notice: the deviations sum to 0. That is why we take the absolute value before averaging.

MAD =

12 ÷ 5

= 2.4

On average, Hùng's monthly goals land 2.4 away from his mean of 4.

STAGE 4 · MEAN ABSOLUTE DEVIATION

Make your prediction

Đức

Goals: 4, 4, 4, 4, 4 · mean 4

MAD =

0

Scored exactly 4 every single month.

Every month is exactly 4, so every distance from the mean is 0.

Hùng

Goals: 6, 8, 2, 0, 4 · mean 4

MAD =

2.4

Some months well above or below 4.

Our worked example.

Tâm *new striker*

Goals: 8, 0, 0, 8, 4 · mean 4

MAD =

?

Big months and 0-goal months. Most are far from 4.

MAD bigger or smaller than Hùng's?

The wager: Will Tâm's MAD be ABOVE or BELOW Hùng's 2.4? Write your answer on your whiteboard, and how sure you are: guess, fairly sure or very sure. Boards up together, before any math.

STAGE 4 · MEAN ABSOLUTE DEVIATION

Your turn: Tâm's MAD

On your own, in your notebook

|x − mean| is the **absolute deviation**: how far x is from the mean. It is never negative.

Goals (x)	Deviation x − mean	Absolute deviation |x − mean|
8		
0		
0		
8		
4		
Sum = ?	**Sum = ?**	**Sum = ?**

Work out:

1. Find the total and the mean.
2. Fill the table.
3. MAD = (sum of |x − mean|) ÷ 5

Check: the deviations in the middle column should add up to 0.

Finished early?
Write one sentence: what does Tâm's MAD tell you about his goals?

STAGE 4 · MEAN ABSOLUTE DEVIATION

Check your work: Tâm's MAD

Total = 8 + 0 + 0 + 8 + 4 = 20 **Mean = 20 ÷ 5 = 4**

Goals (x)	Deviation x − mean	Absolute deviation |x − mean|
8	+4	4
0	−4	4
0	−4	4
8	+4	4
4	0	0
Sum = 20	**Sum = 0**	**Sum = 16**

Check your own table

1. My deviations add up to 0.
2. No absolute deviation is negative.
3. I divided by 5.

Tâm's MAD = 16 ÷ 5

= 3.2

Tâm's MAD is 3.2, above Hùng's 2.4. ABOVE was the right prediction.

STAGE 4 · INTERPRETING MAD

What does MAD tell us?

Đức

MAD =

0

Scored exactly 4 every month. Zero variability in these five months.

Hùng

MAD =

2.4

Moderate spread. Some months far from 4, some close to it.

Tâm

MAD =

3.2

High spread. Goals pile at the extremes: least consistent.

Which striker was most consistent so far? Which was least? Why?

Discuss

You can start: "___ was the most consistent, because his MAD is ___."

STAGE 4 · INTERPRETING MAD

Making sense of MAD

Striker	Goals each month	Mean	Range	MAD
Đức	4, 4, 4, 4, 4	4	0	**0**
Hùng	6, 8, 2, 0, 4	4	8	**2.4**
Tâm	8, 0, 0, 8, 4	4	8	**3.2**

Tâm and Hùng have the same mean and the same range. Why is Tâm's MAD higher?

Use the numbers in the table. Think alone first. Then discuss with your partner.

Why does MAD use $|x - \text{mean}|$, not $x - \text{mean}$?

Hùng's deviations (x − mean): +2, +4, −2, −4, 0. What do these deviations add up to? What would their average be?

In one sentence: what does MAD tell you about Hùng and Đức that the mean alone cannot?

STAGE 4 · INTERPRETING MAD · YOUR CHOICE

What does the MAD tell you?

Pick one card with your partner. Discuss it, then write 1 or 2 sentences that use the MAD.

A · Football

Khánh: 5, 4, 5, 6, 5
mean 5 · MAD 0.4

Sơn: 9, 1, 2, 8, 5
mean 5 · MAD 2.8

What does each MAD tell you about these five months? What tradeoff do you see? What else would you want to know before choosing a striker?

B · Free Fire

Vy: 4, 5, 3, 4, 4
mean 4 · MAD 0.4

Khoa: 9, 0, 1, 8, 2
mean 4 · MAD 3.6

Both players average 4 kills per match. What does the MAD tell you about their matches? What tradeoff do you see? What else would you want to know?

D · Is a low MAD always good?

Đức: 4, 4, 4, 4, 4
mean 4 · MAD 0

Another striker: 0, 0, 0, 0, 0
mean 0 · MAD 0

Both players have MAD = 0. Does that mean they are equally good? What does MAD tell you, and what does it not tell you?

Discuss

STAGE 5 · THE CALLBACK

Hùng or Đức: the same choice now?

At the start, you chose Hùng or Đức from their goals alone. Find what you wrote.

Now you know: **Hùng**: mean 4, MAD 2.4 · **Đức**: mean 4, MAD 0

Would you make the same choice now? Use the statistics to explain.

Write one sentence on your own. Then discuss with your partner.

Discuss

STAGE 6 · FULL TIME · THE CORE LESSON ENDS HERE

Exit slip: every scout signs their name

Complete the exit slip on your own.

1. Find the mean and the MAD of a new striker's goals.
2. Say what the MAD tells you about his goals.
3. Say why the mean is not enough to compare two strikers.
4. Use the mean and the MAD to choose a striker.
5. Optional: give one reason to be careful with only five months of data.

Hand it in at the door.